# ABSTRACT

**of the thesis « Improving the mathematical training of students through learning to solve inverse and ill-posed problems in a pedagogical university», submitted for Doctor of Philosophy (PhD) degree by Akimzhan Nagima on 6D010900 – Mathematics specialty**

**Research relevance.** The main principles of the state educational policy of the country are laid down in the Law of the Republic of Kazakhstan «On Education», in the State program for the development of education and science of the Republic of Kazakhstan for 2016-2019, in the addresses by the President of the Republic of Kazakhstan N.A. Nazarbayev to the People of Kazakhstan: «Strategy «Kazakhstan – 2050»: new political course of the established state», the Plan of the nation - 100 concrete steps to implement five institutional reforms, «Kazakhstan in the new global reality: growth, reform, development», «New conditions of the fourth industrial revolution» and in other regulatory documents.

The main objectives of the lifelong education system in the Republic of Kazakhstan are: improving the competitiveness of education and science, developing human capital for sustainable economic growth, providing economic sectors with competitive personnel with higher and postgraduate education, integrating education, science and innovation, and other major program objectives.

In this regard, a key benchmark in the modern system of higher pedagogical education is to improve the quality of knowledge, skills and professional skills of graduates based on preserving the fundamental nature of education in accordance with the current and future needs of the individual, society and the state.

The development of the components of the mathematical education system is facilitated by the influence of various regularities. These include the priority of scientific research, organized at the intersection of various sciences, the success of which largely depends on the availability of fundamental knowledge.

The need to enhance the quality of teaching mathematical and natural sciences at all levels of education was emphasized by the President of the Republic of Kazakhstan N. Nazarbayev.

One of the important tasks of the fundamentalization of mathematical education is to overcome the historically arisen separation of the natural science and humanitarian components of culture through their mutual enrichment and the search for the foundations of an integral culture at the new stage of civilization development.

The development of the components of the system of mathematical education is influenced by various factors, such as the priority of scientific research organized at the interface of various sciences, the success of which largely depends on the availability of fundamental knowledge; informatization of education, which is a field of scientific and practical human activity aimed at applying methods and means of collecting, storing, processing and transmitting information to systematize available ones; the formation of new knowledge based on the achievement of the psychological and pedagogical goals of training and education of students.

The driving forces behind the development of mathematics have two main objectively existing sources. One of them is connected with the need to systematize the existing mathematical facts, combine them into a coherent theory, develop it, develop methods for solving mathematical problems, and the second with the need to solve problems of natural science, economics, etc. with mathematical tools. Two directions in the development of mathematics corresponding to these two sources are called, respectively, pure mathematics and applied mathematics.

One of the modern directions of applied mathematics is the solution of problems of natural sciences, economics, industry, etc., using the methods of mathematics. Of interest is the inverse and ill-posed problems (hereinafter - IIP).

The applied problems of physics, geophysics, seismology, marine natural disasters, chemistry, photo processing, medicine, economics, ecology, industry, astronomy, astrophysics and other fields can be successfully studied using the theory and methodology of the IIP. IIP are widely used in applied mathematics, in such areas as algebra, analysis, geometry, ordinary differential equations, partial differential equations, integral equations, operator equations, optimal control and other sections of applied mathematics.

At this regard, at the present time in many higher educational institutions of the Commonwealth of Independent States countries, elective courses on IIP are taught on physical, mathematical and natural-science directions of preparation for bachelors and undergraduates. However, elective courses are read in different ways, since there is no single content of the courses of the IIP.

All this requires the study of the scientific and educational potential of the teaching IIP and the research aimed at the creation of scientifically sound methodological training systems for the relevant training courses of the IIP, which are currently lacking in the field of pedagogy and the methodology for teaching mathematics.

All this requires a research approach to the identification of the scientific and educational potential of teaching to solve IIP and formation of a science-based method for organizing the training of students to solve such problems.

The relevance of the given problem determined the research theme: «Improving the mathematical training of students through learning to solve inverse and ill-posed problems in a pedagogical university».

**Research aim** – to develop a scientifically based method of organizing the training of students-mathematicians for solving inverse and ill-posed problems.

**Research object:** system of mathematical training of students at pedagogical university.

**Research subject:** improvement of mathematical training of students through training to solve inverse and ill-posed problems.

**Research hypothesis:** if students are taught to solve inverse and ill-posed problems according to the developed scientifically-based methodology, taking into account the educational potential of inverse and ill-posed problems, this will contribute to the fundamentalization of mathematical training, improve the quality of mathematical and professional training of students of a pedagogical university.

In accordance with the purpose, subject and hypothesis of the research, the

following research objectives are set:

- identify the role and place of inverse and ill-posed problems in the mathematical preparation of students of a pedagogical university;
- analyze the work on learning to solve inverse and incorrect problems at school and university;
- to study the ways of implementing didactic principles in teaching students to solve inverse and ill-posed problems in a pedagogical university;
- determine the content, methods and forms of organization of training for solving inverse and ill-posed problems;
- develop a methodology for teaching inverse and incorrect tasks using information technologies;
- organize and conduct a pedagogical experiment to test the research hypothesis.

**Research methods:** analysis of domestic and foreign scientific works on pedagogy, psychology, philosophy, inverse and ill-posed tasks; analysis of curricula, manuals, theses, conference materials; generalization of experience in teaching the solution of inverse and ill-posed problems; the use of empirical methods of scientific knowledge, such as conversation, observation, conducting lectures and practical exercises with students; pedagogical experiment and analysis of experimental activity.

**Methodological basis of the research** includes fundamental works in the field of

- fundamentalization of education (A.E. Abylkasymova, A.A. Adannikov, E.Y. Bidajbekov, M.A. Bektemesov, N.T. Danaev, I.V. Egorchenko, L.S. Jolgina, S.I. Kalinin, T.Sh. Kal'menov, A.N. Kolmogorov, V.S. Kornilov, L.D. Kudrjavcev, M.A. Lavrent'ev, M.O. Otelbaev, V.G. Romanov, Sh.S. Smagulov, S.L. Sobolev, U.M. Sultangazin, Zh. Sulejmenov, S.E. Temirbolat, A.N. Tihonov, A.Ja. Hinchin, etc.);
- general didactical principles and criteria to optimize the organization of training (Ju.K. Babanskij, V.P. Bespal'ko, V.I. Zagvjazinskij, V.S. Il'in, B.C. Lednev, Usova, etc.);
- problems of informatization of education (K.T. Aldiyarov, Ye.Y. Bidaibekov, T.A. Boronenko, S.G. Grigor'yev, V.V. Grinshkun, A.P. Ershov, G.B. Kamalova, A.A. Kuznetsov, Zh.K. Nurbekova, A.S. Semenov, E.S. Polat, I.V. Robert, A.N. Tikhonov, etc.);
- problem of realization interdisciplinary links (K.T. Aldijarov, E.Y. Bidajbekov, T.A. Boronenko, S.G. Grigor'ev, V.V. Grinshkun, A.P. Ershov, G.B. Kamalova, A.A. Kuznecov, A.S. Semenov, E.S. Polat, I.V. Robert, A.N. Tihonov, etc.);
- inverse and ill-posed problems (A.V. Baev, M.A. Bektemesov, E.Y. Bidajbekov, A.L. Buhgejm, P.N. Vabishevich, A.M. Denisov, K.T. Iskakov, S.I. Kabanihin, T.Sh. Kal'menov, G.B.Kamalova, A.L. Karchevskij, M.M. Lavrent'ev, B.G.Mukanova, S.T.Muhambetzhanov, K.T. Mynbaev, D.B.Nurseitov, A.T. Nurseitova, M.O.Otelbaev, Ju.P. Petrov, A.I. Prilepko, V.G. Romanov, V.S. Sizikov,

Sh.S.Smagulov, U.M. Sultangazin, V.N. Tanana, S.E.Temirbolat, A.N. Tihonov, V.G. Jahno, etc.);

- methodological aspects of the usage of information technologies at university when teaching natural science disciplines (I.V. Belenkova, D.P. Goloskokov, E.A. Daher, S.A. D'jachenko, E.G. Makarov, D.Je. Penni, M.G. Semenenko, V.S. Sizikov, Ju.Ju. Tarasevich, Ch.G. Jedvards, etc.).

**Scientific novelty of the research:**

- the role and place of inverse and ill-posed problems in the mathematical training of students of a pedagogical university have been identified;

- the scientific and educational potential of teaching inverse and ill-posed problems was determined;

- a scientifically grounded methodology for teaching solving inverse and ill-posed problems was developed, which promotes the formation of fundamental knowledge among students in the field of applied mathematics and the skills to apply theoretical knowledge in practice.

**The theoretical significance** of the research lies in the development of methodological and scientifically-methodical foundations of teaching the solution of inverse and ill-posed problems, in identifying and concretizing the scientific and educational potential of teaching inverse and ill-posed problems, and its role in shaping students' fundamental knowledge in mathematics; in the development of methods of using information technology in solving applied problems.

**The practical significance** of the obtained results is that:

- developed a methodology for teaching the solution of inverse and ill-posed problems of students of a pedagogical university;

- describes the methods of rational reasoning used in teaching the solution of inverse and ill-posed problems: hypothesizing an analytical or numerical decision of the IIP, clarifying the course of solving the IIP, controlling convergence and error of a constructive computational algorithm of solving the IIP, sensible analogies when solving the IIP, understanding the physical properties of the object under study in the process of solving IIP and others;

- recommendations on the use of the Mathcad computer math package in laboratory classes in the process of solving educational inverse and ill-posed problems were developed;

- the results and recommendations obtained in the course of the study can be used in teaching mathematical subjects, in writing textbooks on mathematical courses for students and university professors.

**The reliability of the results** of the dissertation research was ensured by the consistency of logical conclusions in the course of a theoretical analysis of the research problems and their consistency with the concepts of applied and pedagogical sciences and fundamental compliance with the main results of other researchers; clarity of methodological, mathematical, historical, mathematical, psychological, pedagogical, didactic and methodological positions; correct application of the systemic, activity, cultural and historical approaches to the problem; using known methods of researching inverse and ill-posed problems; taking into account the experience of colleagues at work, using information

technologies inverse and ill-posed problems, improving the quality of training and the characteristics of students' personal development.

**The basic principles offered for the defense:**

1. Target installations and the scientific and educational potential of learning to solve inverse and ill-posed problems.

2. Methodical system of training in solving inverse and ill-posed problems of students of a pedagogical university.

3. The results obtained in the course of the application and testing of an experimental methodological system for teaching the solution of inverse and ill-posed problems of students of a pedagogical university.

**The results of the research** are introduced into the educational process of the Abai Kazakh National Pedagogical University.

**Approbation of the research results.** The research results have been reported and discussed at the sixth international scientific conference and young scientists school «Theory and computational methods for inverse and ill-posed problems» (Almaty, Kazakhstan, 2014), at the VII international scientific and methodical conference «Mathematical modeling and information technologies in education and science» (Almaty, Kazakhstan, 2015), at the seventh international scientific conference and young scientists school «Theory and computational methods for inverse and ill-posed problems» (Novosibirsk, Russia, 2015), at the international conference on advancements in mathematical sciences (AMS) (Antalya, Turkey, 2015), at the scientific and methodological seminar of the Institute of Mathematics, Informatics and Natural Sciences of the Moscow Autonomous Educational Institution of Higher Education of Physics and Informatics of the Kazakh National Pedagogical University named after Abai (2014–2016).

**Publications on the research results.** The main results of the study were published in 14 scientific papers, including 3 in publications recommended by the Committee for the Control of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, 1 in the journal included in the Scopus database, 2 in scientific journals, 8 - in materials of international conferences, of which 1 - in materials of foreign conferences.

**The structure of the thesis.** The logic of the study and presentation of its results determined the structure of the thesis, consisting of an introduction, two sections, conclusion, list of references and applications.

**References**: In the process of the study 170 references are used.

**Appendix** presents the algorithm of the «hoop» method in finding the roots of polynomials, the acts of introducing the results of research into the educational process of the university.