ANNOTATION

to the thesis of **Alimbekova Nurlana Baurzhanovna**
submitted for the degree of Doctor of Philosophy (PhD)
in the specialty 6D060100 – Mathematics

Research topic: Finite element methods for solving initial boundary value problems for fractional differential filtration equations.

The purpose of the study: Construction and theoretical study of finite element methods for solving initial boundary value problems of filtration in fractured porous media containing fractional time derivatives.

Research objectives.

– Construction of discrete schemes for the approximate solution of the filtration problem in fractured porous media based on the use of finite difference approximation of time derivatives and the finite element method in the spatial direction.

– Construction of high-order discrete schemes for an approximate solution of a two-dimensional filtration problem with a transient filtration law.

– Construction of high-order discrete schemes for an approximate solution of a two-dimensional filtration problem in fractured porous media under the assumption of the presense of two continua.

– Study of the uniqueness of the solution to the considered filtration problems and the continuous dependence of the solution on input data in differential form.

– Stability analysis for the proposed discrete schemes with respect to the input data and the right-hand side of the equation.

– Convergence analysis for the proposed discrete schemes by the method of a priori estimates and determination of the dependence of the convergence order on the order of fractional derivatives.

– Verification of the convergence order and approbation of the proposed discrete schemes for predicting fluid flow in fractured porous media based on computational experiments.

Research methods. Finite element methods, the method of a priori estimates, the methods of functional analysis, the method of computational experiments were used when deriving the results of the thesis.

The main provisions (proven scientific hypotheses and other conclusions that are new knowledge) submitted for defense. Based on the results of the study, the following provisions are submitted for defense:

1) Stable finite element schemes for the numerical solution of the fractional differential problem of filtration in fractured porous media are constructed. The uniqueness of the solution and its continuous dependence on input data, the convergence of finite element schemes are proved.

2) High-order stable finite element schemes are constructed for the numerical solution of a nonlinear fractional differential filtration problem with a transient filtration law. The uniqueness of the solution and its continuous dependence on input data, the convergence of finite element schemes, the convergence of the iterative process is proved, and sufficient conditions for its quadratic convergence are obtained.

3) Stable high-order finite element schemes are constructed for the numerical solution of the fractional differential problem of filtration in fractured porous media under the assumption of the presense of two continua. The uniqueness of the solution and its continuous dependence on input data, the convergence of finite element schemes are proved.

Main results of the study.

1. The initial boundary value problem for the fractional differential equation of filtration in fractured porous media is considered under the assumption that fractures are distributed uniformly over the volume on average. Four special cases of the problem are considered depending on the orders of fractional derivatives. The uniqueness of the solution and its continuous dependence on input data are proved. Finite element methods for solving the problem are constructed. The method of a priori estimates is used to rigorously prove their stability with respect to the initial data and the right-hand side of the equation, the convergence of the approximate solution to the solution of the original differential problem is proved, and the convergence order of the constructed computational schemes is determined. Numerous computational experiments have been carried out to confirm the theoretical convergence order of the schemes. One of the proposed computational methods is applied to the study of the filtration process in a fractured porous medium. It was concluded from the results of computational experiments that the porous medium has a decelerating effect on the flow process, and the orders of fractional derivatives determine the degree of memory influence on the flow behavior.

2. The initial boundary value problem for the nonlinear fractional differential filtration equation, which describes the transient (nonstationary) regime, is considered. The uniqueness of the solution and its continuous dependence on the input data are proved. Newton's iterative method is constructed for solving the problem based on the finite element approximation in the spatial direction and the high-order finite difference method. The method of a priori estimates is used to rigorously prove the stability of the method with respect to the initial data and the right-hand side of the equation, the convergence of the approximate solution to the solution of the original differential problem is proved, and the convergence order of the constructed computational schemes is determined. The convergence of Newton's iterative method is investigated, and sufficient conditions for its quadratic convergence are determined. Computational experiments were carried out to confirm the theoretical convergence order for the proposed scheme.

3. An initial boundary value problem for a fourth-order with respect to the spatial variable fractional differential filtration equation in fractured porous media is considered under the assumption of the presence of two continua, i. e. a system of fractures and a matrix. The uniqueness of the solution and its continuous dependence on input data are proved. A high-order finite element method for solving the problem is constructed. The method of a priori estimates is used to rigorously prove its stability with respect to the initial data and the right-hand side of the equation, the convergence of the approximate solution to the solution of the original differential problem is proved, and the convergence order of the constructed numerical schemes is determined. Numerous computational experiments have been carried out to confirm the theoretical convergence order for the proposed scheme.

Substantiation of the novelty and importance of the results obtained:

The first result is new since numerical schemes have been proposed for solving the fractional differential problem of filtration in fractured porous media under the assumption that fractures are distributed on average uniformly over the volume and theoretical studies have been carried out for the first time. The theoretical convergence orders of schemes have been confirmed by numerous computational experiments. It was concluded from the results of computational experiments that the model under consideration reproduces the characteristic features of the process of fluid flow in a fractured porous medium.

The second result is new since an iterative Newton method based on a finite element approximation in the spatial direction and a high-order finite difference method in the time variable was proposed for solving a nonlinear fractional differential filtration problem with a transient (nonstationary) regime, and theoretical studies were carried out for the first time.

The third result is new since a higher-order numerical scheme was proposed for solving a fractional differential problem of filtration in fractured porous media containing fourth-order spatial derivatives under the assumption of the presence of two continua, a system of fractures and a matrix, and theoretical studies were carried out for the first time.

Compliance with the directions of science development or state programs:

«A fair state. One nation. Prosperous society» address of the head of state Kassym-Jomart Tokayev to the people of Kazakhstan (September 1, 2022); Concept for the development of Information and communication technologies and the digital sphere (№961, December 30, 2021); National development plan of the Republic of Kazakhstan until 2025 (№521, February 26, 2021).

Description of the contribution of the doctoral student to the preparation of each publication:

27 scientific works were published on the topic of the thesis including 3 papers in foreign journals included in the Scopus and Web of Science databases, 7 papers in scientific publications approved by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan, 14 publications in the proceedings of International conferences, 1 textbook, 2 copyright certificates to software for carrying out computational experiments on solving filtration problems by the finite element method. All publications were prepared in the course of the study.

1. Convergence Analysis of a Numerical Method for a Fractional Model of Fluid Flow in Fractured Porous Media // Mathematics. — 2021. — Vol. 9, No. 2179. — P. 1–24. (Co-authors: Baigereyev D., Berdyshev A., Madiyarov M., 80%) https://www.mdpi.com/2227-7390/9/18/2179. In the paper, the doctoral student constructed and investigated numerical methods for solving the problem for the fractional differential equation of filtration in fractured porous media.

2. A Priori Estimates for the Solution of an Initial Boundary Value Problem of Fluid Flow through Fractured Porous Media // Axioms. — 2022. — Vol. 8, No. 408. — P. 1–20. (Co-authors: Berdyshev A., Baigereyev D., 90%) https://www.mdpi.com/2075-1680/11/8/408. In the paper, the doctoral student constructed and studied a numerical scheme for a filtration model that includes a nonlinear equation containing several terms with fractional derivatives in the Caputo sense of order (1, 2).

3. Galerkin approximations for an initial boundary problem of transient flow in fractured porous media // Lobachevskii Journal of Mathematics. – 2022. – Vol. 43, No. 11. – P. 3048–3056. (Co-authors: Baigereyev D., Berdyshev A., 80%) https://link.springer.com/article/10.1134/S1995080222140049. In this paper, a theoretical estimate for the solution of an approximation problem for solving a fractional-differential problem for a fourth-order filtration equation in fractured porous media is obtained under the assumption of the existence of two continua.

4. Numerical solution of a fractional order differential equation // Bulletin of KazNPU. Physical and mathematical sciences. — 2019. — No. 4 (68). — P. 18–25. (Co-authors: Berdyshev A. S., Baigereyev D. R., 80%) https://bulletin-phmath.kaznpu.kz/index.php/ped/issue/view/7/5 (in Russian). In this work, the doctoral student constructed and studied a numerical method for the Dirichlet problem for a fractional order partial differential equation.

5. Investigation of a numerical method for solving a boundary value problem for a differential equation with a fractional time derivative // Izvestiya of Altai State University. — 2020. — Vol. 114, No. 4. — P. 64–69. (Co-authors: Baigereyev D. R., Madiyarov M. N., 80%) http://izvestiya.asu.ru/article/view/%282020%294-10 (in Russian). In this study, the doctoral student constructed and investigated a numerical method for solving an equation with a fractional time derivative in the sense of Caputo-Fabrizio.

6. Study of the initial boundary value problem for a two-dimensional convection-diffusion equation with a fractional time derivative in the sense of Caputo-Fabrizio // Bulletin of KazNU. Series of Mathematics, Mechanics, Computer Science. — 2021. — No. 2 (110). — P. 113–127. (Co-author: Oskorbin N. M., 95%) https://bm.kaznu.kz/index.php/kaznu/article/view/937. In the work, the doctoral student investigated a numerical method for solving an initial boundary value problem for a differential equation with a fractional derivative with respect to time in the sense of Caputo-Fabrizio.

7. Numerical implementation of a nonlinear model of fluid flow in a highly fractured medium by the finite element method // Bulletin of the NEA RK. — 2021. — No. 3 (81). — P. 8–17 (100%) https://journal.neark.kz/chislennaya-realizacziya-nelinejnoj-modeli-filtraczii-v-silno-treshhinovatoj-srede-metodom-konechnyh-elementov/. The work of the doctoral student presents the results of a study of an iterative method for solving a nonlinear filtration problem in highly porous fractured reservoirs.

8. Analysis of the numerical solution of the three-phase nonisothermal fluid flow problem // Bulletin of NEA RK. — 2022. — No. 2 (84). — P. 145–155. (Co-author: Baigereyev D. R., 90%) https://journal.neark.kz/analiz-chislennogo-resheniya-zadachi-trehfaznoj-neizotermicheskoj-filtraczii/. The doctoral student constructed a numerical method for solving the three-phase non-isothermal filtration problem, taking into account capillary forces, and studied its stability and convergence.

9. Error estimates of the numerical method for the filtration problem with Caputo-Fabrizio fractional derivatives // Bulletin of KazNU. Series Mathematics, Mechanics, Computer Science. — 2022. — No. 2 (114). — P. 101–116. (Co-authors: Baigereyev D. R., Oskorbin N. M., 90%) https://bm.kaznu.kz/index.php/kaznu/article/view/998. In the paper, the doctoral student proposed a numerical method for solving the filtration problem with fractional Caputo-Fabrizio derivatives and theoretically investigated the order of its convergence.

10. Finite element method for solving a fractional flow model in porous media // Bulletin of KazNPU. Physical and mathematical sciences. — 2022. — No. 1 (77). — P. 7–14. (Co-authors: Baigereyev D., Berdyshev A., 90%) https://bulletin-phmath.kaznpu.kz/index.php/ped/article/view/597. The doctoral student considered the filtration equation for a viscoelastic fluid in a fractured porous medium with fractional time derivatives in the sense of Caputo.

11. Study of initial boundary value problem for two-dimensional differential equation with fractional time derivative in the sense of Caputo // Third International Conference on Material Science, Smart Structures and Applications: (ICMSS 2020). — 2021. — P. 1–6. (Co-authors: Berdyshev A. S., Baigereyev D. R., 80%).

12. Parallel Implementation of the Algorithm for Solving a Partial Differential Equation with a Fractional Derivative in the Sense of Riemann-Liouville // 2021 IEEE International Conference on Smart Information Systems and Technologies (SIST). — 2021. — P. 1–6. (Co-authors: Berdyshev A. S., Baigereyev D. R., 80%).

13. Simulation of the air pollution process of an industrial city // Lomonosov Readings in Altai: Fundamental Problems of Science and Technology: Collection of scientific art. intl. conf.–– 2018. — P. 620–623. (Co-authors: Madiyarov M. N., Yergaliyev Ye. K., 80%) (in Russian).

14. Modeling of processes based on fractional partial differential equations // All-Russian Scientific and Practical Conference: Mathematics-Altai Territory (MAK-2019). — 2019. — P. 234–237. (Co-authors: Berdyshev A. S., Madiyarov M. N., 80%). (in Russian).

15. On a method for the numerical solution of a differential equation of fractional order // Collection of mater. of Int. scientific-practical conf. «Uly Dala Ustazy». — 2020. — P. 292–296. (Co-author: Baigereyev D. R., 80%). (in Russian).

16. Numerical solution of diffusion-convection equations with fractional time derivative // Creativity of the youngs is innovative development of Kazakhstan: Materials of the VI Intern.

sci.-tech. conf.— 2020. — P. 36–39. (Co-authors: Baigereyev D. R., Yergaliyev Ye. K., 80%). (in Russian).

17. Construction of methods for solving the filtration problem in highly porous fractured reservoirs // All-Russian Scientific and Practical Conference: Mathematics-Altai Territory (MAK-2020). — 2020. — P. 3–7. (Co-authors: Baigereyev D. R., Yergaliyev Ye. K., 80%). (in Russian).

18. Finite element method for solving a fractional flow model in porous media // Joint issue of journals «Bulletin of the NEA RK» and «Computational Technologies» based on the materials of the international conference «Computing and Information Technologies in Science, Technology and Education» (CITech-2020). — 2020. — No. 3 (1). — P. 48–53. (Co-authors: Baigereyev D., Yergaliyev Ye., 80%).

19. Numerical solution of the wave-diffusion equation with the fractional Caputo derivative // All-Russian Scientific and Practical Conference: Mathematicians to the Altai Region (MAK-2021). — 2021. — P. 76–80. (Co-authors: Baigereyev D. R., Салыков Р. М., 80%). (in Russian).

20. Finite-element method for solving the problem of three-phase non-isothermal filtration of a compressible liquid, taking into account phase transitions // All-Russian Scientific and Practical Conference: Mathematicians to the Altai Region (MAK-2022). — 2022. — P. 211–215. (Co-author: Baigereyev D. R., 80%). (in Russian).

21. Numerical methods for fractional models of fluid flow in fractured porous media // Abstract of the Uzbekistan-Malaysia International Conference “Computational Models and Technologies (CMT2022)”. — 2022. — P. 24–25. (Co-authors: Baigereyev D., Berdyshev A., 80%).

22. Parallel implementation of the algorithm for solving the problem of fluid flow in fractured porous media // Abstract of the Uzbekistan-Malaysia International Conference “Computational Models and Technologies (CMT2022)”. — 2022. — P. 23–24. (Co-author: Baigereyev D., 80%).

23. Theoretical estimation of parallel algorithms for problems of multiphale flow in porous media // Abstract book of the International Conference «Computational and Information Technologies in Science, Engineering and Education (CITech-2022)». — Almaty, 2022. — P. 10. (Co-authors: Baigereyev D.R., Baishemirov Z.D., 80%).

24. On Fractional-Differential Filtration Models and Basic Approaches to the Construction of Projection Methods for Their Implementation // Collection of Mater. Intern. scientific-pract. conf. «Uvaliyev readings-2022» «Actual problems of science and education in the face of modern challenges».— 2022. — P. 17–22. (Co-authors: Baigereyev D. R., Berdyshev A. S., 90%). (in Russian).

25. Finite element methods for solving filtration problems: textbook. — Ust-Kamenogorsk: S. Amanzholov EKU «Berel» publishing house, 2022. — P. 109. (Co-author: Baigereyev D. R., 70%). (in Kazakh)

26. Software package for parallel implementation of the fractional differential filtration model on graphic processors (software). — 2022. (Co-authors: Baigereyev D. R., Yergaliyev Ye. K., Omarieva D. A., Boranbek K., 70%).

27. Software package for carrying out computational experiments on solving problems of filtering by the finite element method (software). — 2022. (Co-author: Baigereyev D. R., 70%).